Math 53: Multivariable Calculus **Sections 102, 103**

Worksheet for 2020-04-17

Conceptual Review

Question 1. In the below, f, g denote scalar fields while $\mathbf{F}, \mathbf{G}$ denote vector fields. Which of these expressions do not make sense? For the ones that do make sense, do they output a scalar field or a vector field?

(a) $\operatorname{grad} f$
(b) $\operatorname{curl}(\operatorname{div} \mathbf{F})$
(c) $\operatorname{grad}(\operatorname{div}(\operatorname{curl} \mathbf{F}))$
(d) $\nabla(\nabla \times f)$
(e) $\nabla \cdot (\nabla \times (f\mathbf{G}))$
(f) $\nabla \cdot (\nabla f \times \nabla g)$

Question 2. Of the expressions that do make sense above, are there any that are *always* equal to zero (either the zero scalar field or the zero vector field)?

Hint: The vector identity $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot (\nabla \times \mathbf{F}) - \mathbf{F} \cdot (\nabla \times \mathbf{G})$ may be useful for one of the expressions.

Problems

Problem 1. Let n be a constant, let r denote the scalar field $\sqrt{x^2 + y^2 + z^2}$, and let $\mathbf{r}$ denote the vector field $\langle x, y, z \rangle$.

Compute the following:

(a) $\nabla(r^n)$
(b) $\nabla \times (r^{n-1}\mathbf{r})$
(c) $\nabla \cdot (r^{n-1}\mathbf{r})$.

For part (c), what value of n makes the answer equal to zero?